GNSS Solutions:

GPS jamming and linear carrier phase combinations

"GNSS Solutions" is a regular column featuring questions and answers about technical aspects of GNSS. Readers are invited to send their questions to the columnist, **Dr. Mark Petovello**, Department of Geomatics Engineering, University of Calgary, who will find experts to answer them. His e-mail address can be found with his biography at the conclusion of the column.

What is the effect of GPS jamming on maritime safety?

Although GPS jamming incidents are relatively rare they can occur; and when they do, their impact can be severe.

The General Lighthouse Authorities of the United Kingdom and Ireland (GLAs) comprise the Commissioners of Irish Lights, the Commissioners of Northern Lighthouses and Trinity House, who between them provide aids to navigation (AtoNs) for the benefit of all mariners in British and Irish waters. In order to investigate the effects of GPS jamming, whether by intentional or accidental means, the GLAs conducted a trial in 2008 on the effect of GPS denial on marine aids-to-navigation, and ship-borne and shore-based navigation and information systems.

Today's mariners commonly use GPS enabled devices to navigate their vessels, however large, from port to port and berth to berth. The International Maritime Organization (IMO) mandates the carriage of electronic position-fixing systems by all vessels over 300 gross tons and those carrying passengers on an international voyage in accordance with the Safety of Life at Sea (SOLAS) convention. The GPS position is often fed into other vessel systems, for example an electronic chart display and information system (ECDIS), the vessel's automatic identification system (AIS), or a plotter.

The use of differential GPS (DGPS) is preferred; mariners improve their positioning accuracy and ensure integrity of their GPS derived position by using the large number of DGPS radiobeacons located around the world.

Although GPS receivers for navigation are commonplace and very conspicuous on the bridge, the use of GPS is often more inconspicuous in other AtoN and positioning devices. Examples include its use for providing position input to the onboard AIS transponder, as well as the digital selective calling (DSC) system, which has the capability to include the vessel's position as part of a distress signal.

In addition to vessel-based systems, marine aids-to-navigation use GPS. AIS timeslots may be synchronized using GPS as a source of accurate time. AIS also provides AtoN position information based on GPS input. Synchronized lights use GPS as a common timing source, and differential GPS services provide accuracy and integrity to the mariner.

Therefore, GPS denial, whether intentional from malicious jamming or unintentional due to malfunctioning equipment such as television antennas, may affect safety both on the bridge and on-shore.

During the jamming trial, a GLA vessel was fitted with two typical marine-grade DGPS receivers, a survey-grade GPS receiver, and an eLoran receiver. The UK Ministry of Defence assisted in the trial by providing and operating a GPS jamming unit.

The jamming unit transmitted a known pseudorandom noise code on the L1 frequency, with an effective radiated power of 1.5 W. On full power, using a directional antenna, this unit was capable of jamming GPS over a 30-kilometer envelope (**Figure 1**). The trial vessel made several runs between two waypoints.

Each waypoint was positioned outside the jamming area (including the areas affected by the main lobe and side lobes as indicated by the red and black hatching areas in Figure 1) to allow onboard GPS devices to

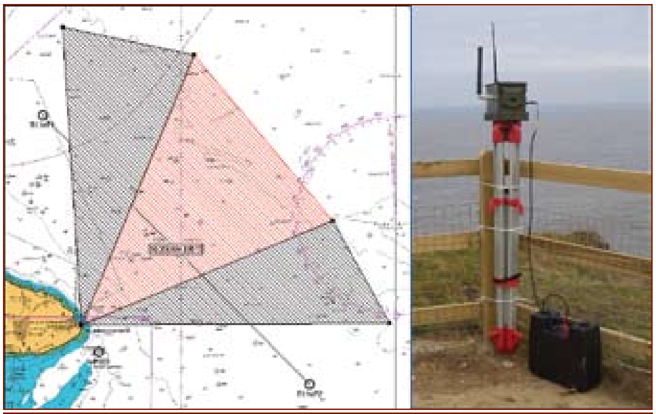

FIGURE 1 On the left is a screen capture showing the area affected by the GPS jamming unit (shown on the right) along with the passage between the waypoints used in the trial, plotted using Meridian SeaTrack software.

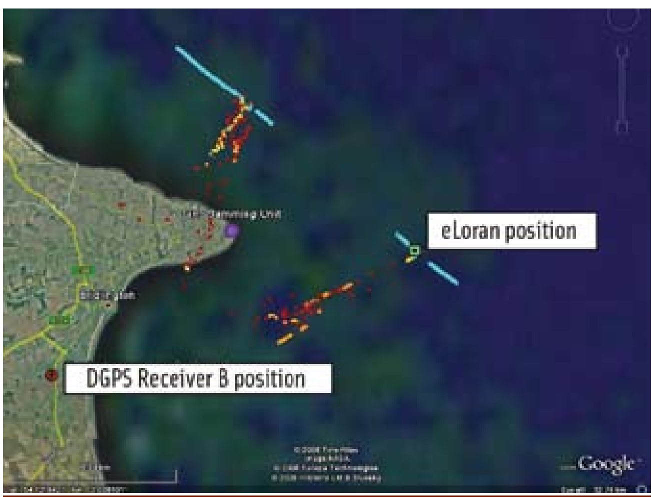

FIGURE 2 Google Earth plot showing reported position from one of the GPS receivers during a passage through the jamming zone. Highlighted is a comparison of the worst-case erroneous GPS position (red circle) with the corresponding eLoran position (green square). Colors indicate reported speed: blue <15 knots, yellow <50 knots, orange <100knots, and red >100knots.

reacquire satellites before starting the next run. Each passage took approximately one hour at a steady speed of 10 knots.

The results were a mixture of the expected and the surprising. We had expected that the GLAs trial eLoran service would not be affected by the GPS jamming unit, due to the different operating frequencies and dissimilar failures modes, and this was indeed the case. eLoran gave a consistent position accuracy of 8.0 meters (95%) during periods of GPS jamming, demonstrating the advantages of an independent electronic navigation system with dissimilar failure modes to GPS.

The typical marine-grade GPS receivers did not fare so well. Both units were forced to operate in stand-alone GPS mode as the local DGPS reference station was also jammed. Both receivers reported erroneous positions, often indicating implausibly high speeds and equally implausible position errors.

Figure 2 details the recorded positions from one of the GPS receivers in which each recorded position has been color-coded depending on the reported speed. Where the receiver is operating correctly, the resulting positions are reported as blue. The effect of GPS jamming can be seen with the yellow, orange and red positions, which are reporting erroneous speeds and positions.

This figure also shows a comparison of the worst-case GPS position with that provided by the eLoran receiver at the same measurement epoch. It can be clearly seen that the two reported positions differ significantly, being over 22 kilometers apart.

Not only shipborne systems were affected. The AIS enables vessels to communicate with other vessels and shore-based infrastructure to exchange information such as their call sign, position, destination, estimated time of arrival, and other pertinent information. This information is often used by vessel traffic systems (VTS) on shore to monitor traffic in and out of port and other waterways. During periods of GPS jamming the traffic picture can be compromised due to

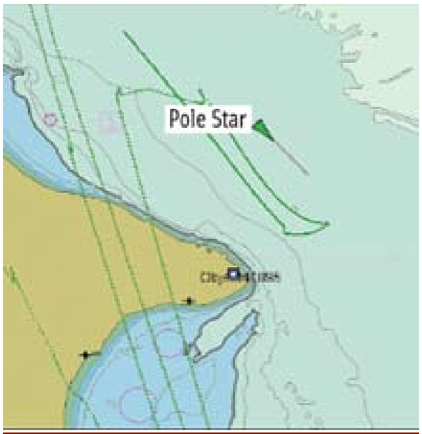

FIGURE 3 Extract of a vessel traffic image during the jamming trial. GPS jamming resulted in erroneous positions being reported for the trial vessel "Pole Star," giving the impression that she has sailed across the peninsula.

erroneous positions being reported by the vessel's GPS receivers (**Figure 3**).

It is also important to note that, during the trial, shipborne systems reliant on GPS input failed to maintain GPS lock and alarmed audibly. This resulted in a higher level of noise on the bridge lasting several minutes and also in the denial of some shipborne systems, such as the vessel's ECDIS, gyro calibration, dynamic positioning system, and input to the DSC. A lack of familiarization of the vessel's crew in such situations could clearly affect their ability to respond, particularly if an outage occurs while the vessel is performing a difficult maneuver.

In reply to the original question posed by the column, the GLA trial demonstrated that some typical marine grade GPS receivers can be affected substantially, reporting erroneous positions, and implausible speeds. In addition, other GPS dependent systems can be adversely affected; the vessel's AIS unit for example. In such a situation the traffic image for the area, whether viewed from other vessels or by shore-based infrastructure, would be seriously confused.

Particularly important to the GLAs was the fact that some AtoNs were affected. DGPS services were disrupted, AIS AtoNs were affected, and synchronized lights were also vulnerable. This was to be expected and now the GLAs are in a position to be able to identify when GPS denial occurs and to be able to respond appropriately.

IMO and the GLAs promote the use of multiple, dissimilar navigation systems, just for this kind of event. As such, the GLAs recommend that all mariners be familiar with diverse navigation systems, and are promoting the use of eLoran as a terrestrial back-up and complementary system to satellite navigation.

ALAN GRANT AND PAUL WILLIAMS

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Paul Williams is a principal engineer with the Research and Radionavigation Directorate of The General Lighthouse Authorities of the UK and Ireland. As the technical lead of the GLA's eLoran Work Program, he is involved in planning the GLA's maritime eLoran trials and works on a wide range of projects from real-time differential-Loran system development to the quality assurance of Loran additional secondary factor (ASF) data. He holds B.Sc. and Ph.D. degrees in electronic engineering from the University of Wales, and is a chartered engineer, an Associate Fellow of the Royal Institute of Navigation, and a board member of the International Loran Association.

What are linear carrier phase combinations and what are the relevant considerations?

Linear carrier phase combinations are formed by adding or subtracting carrier phase measurements on two or more frequencies. Such combinations are used to improve the resulting measurement in some manner relative to the original measurements.

In this context, "improvement" usually implies removing/reducing certain errors so as to facilitate the ambiguity resolution process or increase the measurement (and, therefore, position) precision. We must note, however, that improvement in both areas is not possible and thus a design trade-off is required.

In this "solution," we will discuss how linear carrier phase combinations are formed and the key considerations associated with this process. A discussion of some of the common GPS combinations is also provided.

Linear Combinations

Denoting the carrier phase measurements (in units of cycles) on frequencies f_1 and f_2 as ϕ_1 and ϕ_2 respectively, the linear carrier phase combination, $\phi_{a,b}$, is computed as

$$\phi_{a,b} = a\phi_1 + b\phi_2 \quad (1)$$

where a and b are selectable coefficients. The wavelength of the new measurement is given by

$$\lambda_{a,b} = \frac{c}{af_1 + bf_2} = \frac{\lambda_1\lambda_2}{b\lambda_1 + a\lambda_2} \quad (2)$$

where c is the speed of light, and λ_1 and λ_2 are the wavelengths corresponding with frequencies f_1 and f_2 respectively.

Although equation (1) only considers two measurements, we could also add other measurements (e.g., $c\,\phi_3$, $d\,\phi_4$, etc.). These additional measurements are omitted here for simplicity without detracting from the generality of what follows.

Before proceeding, first consider the equation for a carrier phase measurement made on a single frequency, f_1, which, for our current purposes, can be written as

$$\phi_i = \frac{1}{\lambda_i}(\rho + G + I_i) + N + n \qquad (3)$$

where ρ is the geometric range from the receiver to the satellite, G the combined effect of all "geometric" errors, I_i is the ionosphere error, N is the integer carrier phase ambiguity, and n is the combined effect of the stochastic errors, namely, measurement noise and multipath.

Geometric errors are characterized as being the same magnitude on all frequencies and, in the context of GNSS, include the troposphere and satellite orbit errors (receiver and satellite clock errors can also be included here, but are normally removed during double difference processing). The ionospheric error is given by

$$I_i = \frac{40.3 \times TEC}{f_i^2} = \frac{K}{f_i^2}$$

where TEC is the total electron content (TEC).

Both the geometric and ionospheric errors are included within the brackets implying that they are defined in units of length. The measurement noise, however, is outside the brackets and is quantified in units of cycles. We will explain the reason for this distinction in a moment.

In the context of the foregoing discussion, three main considerations need to be borne in mind when dealing with carrier phase combinations: the integer nature of the ambiguities, the magnitude of the errors in units of cycles, and the magnitude of the errors in units of meters. Each of these is discussed in detail in the following sections.

Integer Nature of the Ambiguities

In order for the ambiguities of the combined carrier phase measurement to be integer, the a and b coefficients must also be integer. Note that this does not preclude using non-integer coefficients. In fact, the well known ionosphere-free combination falls into this category (which we will discuss later). Rather, the key is that if non-integer coefficients are used, the linear combination cannot be used to resolve the ambiguities as integers.

Magnitude of Errors in Units of Cycles

Errors in units of cycles ("cycle-errors") are important because they have direct implications on the ambiguity resolution process. Specifically, when cycle-errors are small, the ambiguity resolution process is more reliable. Conversely, large cycle-errors make ambiguity resolution less reliable.

With this in mind, the various errors are investigated below by substituting equation (3) into equation (1) and considering only the error source

a	b	Common Name	Geometric Errors	Ionosphere Error	Stochastic Errors
1	-1	Widelane	0.22	0.28	1.41
1	1	Narrowlane	1.78	2.28	1.41
1	$-f_2/f_1$	Ionosphere free	0.39	0	1.27
4	-3	N/A	1.66	0.15	5

TABLE 1. Amplification of errors in units of cycles (relative to L1) for some common GPS linear combinations

a	b	Common Name	Geometric Errors	Ionosphere Error	Stochastic Errors
1	-1	Widelane	1	1.28	6.41
1	1	Narrowlane	1	1.28	0.79
1	$-f_2/f_1$	Ionosphere free	1	0	3.23
4	-3	N/A	1	0.09	3.01

TABLE 2. Amplification of errors in units of length (relative to L1) for some common GPS linear combinations

of interest. With this in mind, the geometric cycle-errors are given by

$$\begin{aligned} G_{a,b} &= \frac{a}{\lambda_1}G_1 + \frac{b}{\lambda_2}G_2 \\ &= \frac{b\lambda_1 + a\lambda_2}{\lambda_1\lambda_2}G \\ &= \frac{1}{\lambda_{a,b}}G \end{aligned} \tag{4}$$

where the frequency independent nature of the geometric errors in used in the first step, and equation (2) was used in the last step.

The importance of this result is that the geometric errors are scaled by the new wavelength. In other words, selecting *a* and *b* to obtain a longer wavelength reduces the geometric errors in units of cycles. This makes ambiguity resolution more reliable. Similarly, shorter wavelengths will increase the errors and make the ambiguity resolution process more difficult.

For the ionosphere, the error in the linear combination is

$$\begin{aligned} I_{a,b} &= \frac{a}{\lambda_1}\frac{K}{f_1^2} + \frac{b}{\lambda_2}\frac{K}{f_2^2} \\ &= \frac{a}{\lambda_1}\frac{K}{f_1^2} + \frac{b}{\lambda_2}\frac{f_1^2}{f_1^2}\frac{K}{f_2^2} = \left(\frac{a}{\lambda_1} + \frac{b}{\lambda_2}\frac{f_1^2}{f_2^2}\right)\frac{K}{f_1^2} \\ &= \frac{1}{\lambda_1}\frac{K}{f_1^2}\left(a + \frac{b\lambda_1}{\lambda_2}\frac{f_1^2}{f_2^2}\right) = \frac{1}{\lambda_1}\frac{K}{f_1^2}\left(a + \frac{bf_2}{f_1}\frac{f_1^2}{f_2^2}\right) \\ &= I_1\left(\frac{af_2 + bf_1}{f_2}\right) \end{aligned} \tag{5}$$

where the relation of $c = \lambda f$ was used in the third line.

In this case, the effect of the ionosphere is not explicitly related to wavelength and must be evaluated on the per-combination basis. Instead, the error is expressed as the function of the error on one of the two input measurements. From the preceding discussion, if *a* is set to unity and *b* is set to $-f_2/f_1$, then the effect of the ionosphere error is removed altogether.

This is the well-known ionosphere-free linear combination.

Finally, to assess the stochastic errors, we must employ error propagation. For this, we assume that the measurement errors are a function of the wavelength — this is true for both noise and multipath effects — and that the errors are white and uncorrelated between frequencies. This latter assumption is certainly not valid for multipath effects. Nevertheless, the assumption simplifies the derivation, which can, as necessary, be expanded to consider more realistic multipath effects.

That said, with the stated assumptions, the stochastic errors are quantified by their standard deviation and are given by

$$\sigma_{a,b} = \sqrt{a^2\sigma_1^2 + b^2\sigma_2^2} \tag{6}$$

Table 1 summarizes the effect of some common GPS linear combinations in terms of errors in units of cycles. In particular, the table shows the ratio of the errors for the linear combination to the corresponding error of the L1 measurement. To put it differently, the table shows the "amplification" of the errors relative to L1 (absolute value).

The benefit of the widelane combination ($\lambda \approx 0.86$.m) becomes clear here. In particular, it reduces all of the geometric and ionospheric errors relative to L1, thus simplifying the ambiguity resolution process. In contrast, the narrowlane combination ($\lambda \approx 0.11$ m) increases all errors, suggesting that it should be avoided unless the errors are small. (The benefit of this combination will be investigated in the next section.)

As expected, the ionosphere-free combination removes the ionosphere error and is therefore still important, for example, over very long baselines or during solar maximum when the ionosphere errors are expected to be large. However, we must remember that the ionosphere-free combination does not maintain integer ambiguities. The last combination listed is an integer-maintaining combination that closely approximates the ionosphere-free case in terms of mitigating the ionosphere error, which may be useful in some applications.

It is also interesting to note that in all cases the stochastic errors are increased. We should expect this because, regardless of the values of *a* and *b*, the sum squared formulation in equation (6) guarantees an increase in the variance of the errors.

Magnitude of Errors in Units of Length

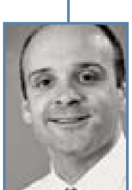
We can determine the magnitude of the various errors in units of length by multiplying their cycle-errors by their wavelength. Errors in units of length are important for positioning purposes.

Effectively, by scaling the carrier phase measurement to units of length, one obtains a range that can be used to compute one's position (similar to the pseudorange case). It follows, therefore, that smaller measurement errors will give correspondingly smaller position errors. Table 2 shows the same comparison as Table 1, but for the case when the errors are expressed in units of length.

Three things are worth noting. First, the geometric errors are unaffected. This is not surprising because these errors, by definition, were invariant to frequency.

Second, the design trade-off discussed earlier is now evident. For example, the widelane combination is shown to increase the measurement error due to the ionosphere by about 28 percent. (The noise is also increased significantly, but this is less of a concern because it can be averaged out.) Somewhat ironically however, the motivation for using the widelane in the first place is usually because the errors — most notably due to the ionosphere — are too large for reliable ambiguity resolution of the L1 ambiguities directly.

Third, the narrowlane combination is shown to have better stochastic error performance relative to L1 when expressed in units of length. This is generally considered to be the advantage of this combination. Specifically, for short baselines (e.g., for attitude determination), where the stochastic errors dominate, the narrowlane combination is preferred. Of course, this comes at the expense of having to resolve the ambiguities for a shorter wavelength — a relatively less reliable approach.



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Summary and Outlook

The foregoing analysis focused on dual-frequency combinations. However, with the modernization of GPS and the upcoming launches of Galileo and Compass, multiple frequency combinations will be possible. Despite this, the considerations discussed in this article will still hold and can be used as a stepping stone for more advanced combinations and subsequent data processing.

MARK PETOVELLO

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