GNSS Solutions:

What are vector tracking loops, and what are their benefits and drawbacks?

"GNSS Solutions" is a regular column featuring questions and answers about technical aspects of GNSS. Readers are invited to send their questions to the columnist, **Dr. Mark Petovello**, Department of Geomatics Engineering, University of Calgary, who will find experts to answer them. His e-mail address can be found with his biography at the conclusion of the column.

Vector tracking loops are a type of receiver architecture. The difference between traditional receivers and those that use vector tracking algorithms is the manner in which they process the received GNSS satellite signals, and how they determine the receiver's position and velocity.

Vector-based tracking loops combine the two tasks of signal tracking and position/velocity estimation into one algorithm. In contrast, traditional — or scalar — tracking methods track each satellite's signal(s) independently; both of each other and of the position/velocity solution.

Vector tracking has many advantages over scalar tracking loops. The most commonly cited advantage is increased immunity to interference and jamming. The minimum carrier-to-noise power density ratio *(C/N_0)* at which the receiver can operate is lowered by processing the signals in aggregate instead of separately.

Vector tracking algorithms also have the ability to bridge signal outages and immediately reacquire blocked signals. Moreover, vector tracking loops have a greater immunity to receiver dynamics than scalar tracking loops.

A final advantage: The vector tracking architecture allows the receiver's motion to be constrained in different dimensions, which can be exploited by receivers whose motion occurs primarily in one or two directions, such as ships or automobiles, for example.

The primary drawbacks of vector tracking loops relative to traditional approaches are their processing load and complexity. The Kalman filter used by the vector tracking architecture (more details to follow) must be iterated on a time scale commensurate with the integrate-and-dump period used by the algorithm (~ 50 Hz). The numerically controlled oscillators (NCOs) in each channel also must be controlled directly by the central Kalman filter.

Another drawback of vector tracking is that the presence of a fault in one channel will affect all the other channels, possibly leading to receiver instability or loss of lock on all satellites.

Before discussing how vector tracking loops operate, let's first review how a traditional receiver operates. **Figure 1** shows a block diagram of a typical GPS receiver.

In the traditional GNSS receiver, scalar tracking loops are used to estimate the pseudoranges and pseudorange-rates for the available satellites. A delay lock loop (DLL) is generally used for estimating the pseudoranges, and either a Costas loop or frequency lock loop (FLL) is used to estimate the pseudorange-rates or carrier Doppler. (A phase lock loop can also be implemented, although it is not strictly required for signal tracking).

The pseudoranges and pseudorange-rates are fed forward to the navigation processor, which solves for the receiver's position, velocity, clock bias, and clock drift (i.e., the navigation states). The navigation processor is typically an iterative least squares algorithm or a Kalman filter.

In Figure 1, note that the flow of information in the receiver is strictly from left to right. Each channel of the receiver tracks its respective signal independent of the other channels. In addition, no information from the navigation processor is fed back to the tracking loops.

The only exception to this may occur when the navigation solution is used to initialize the acquisition process for a particular satellite. Although this may reduce acquisition time, it does not improve the receiver's satellite tracking capability.

By its very nature, the traditional receiver architecture does not exploit the inherent relation between signal

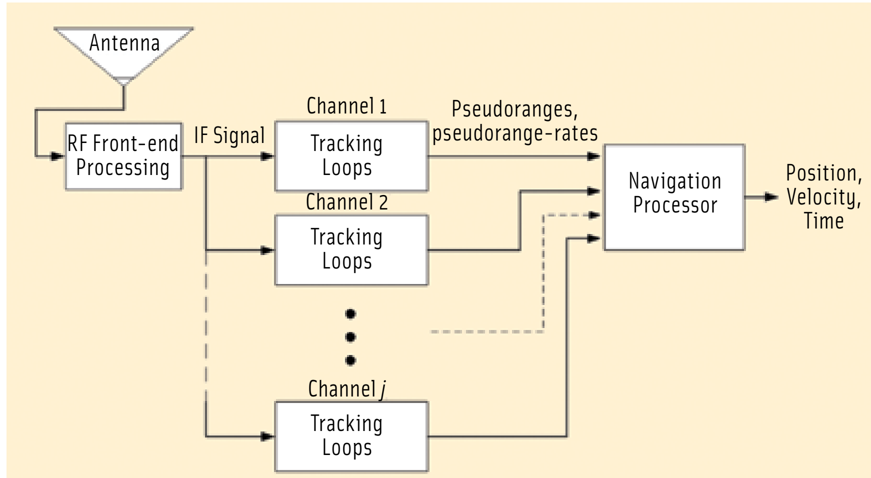

FIGURE 1 **Traditional Receiver Architecture**

tracking and navigation state estimation. In particular, recall that the basic concept of GNSS is that the signal tracking information (i.e., pseudoranges and pseudorange-rates) can be used to estimate the desired navigation states (i.e., position, velocity, and clock information).

In contrast to traditional receivers, vector tracking algorithms exploit the inherent coupling between signal tracking and navigation solution computation, and combines them into a single step. In other words, in a vector tracking approach, the navigation processor is used to perform both tasks and eliminates the need for intermediate tracking loops.

Figure 2 shows a block diagram of a receiver employing a vector delay/frequency lock loop (VDFLL). In this architecture, the pseudoranges and pseudorange-rates are predicted by the navigation processor (in this case an extended Kalman filter (EKF)) for each signal that is to be tracked. This prediction is performed using the estimated navigation states and the computed satellite position and velocity.

Each channel of the receiver then produces pseudorange and pseudorange-rate residuals (differences) relative to the predicted pseudorange and pseudorange-rates. In turn, the EKF uses the residuals to update its estimates of the receiver's navigation states. In the VDFLL, the vector tracking loop is closed through the EKF.

For a VDFLL, the typical states used in the EKF are shown in equation (1).

$$x = \begin{bmatrix} \delta P_{3x1} \\ \delta V_{3x1} \\ \delta B_{2x1} \end{bmatrix} = \begin{bmatrix} \text{Position} \\ \text{Velocity} \\ \text{Clock Bias \& Drift} \end{bmatrix} \quad (1)$$

Higher order derivative states can be appended to (1) but are not necessary for the VDFLL to function. The residuals produced in the *j-th* channel are related to errors in the states of the EKF by equation (2).

$$\begin{bmatrix} \delta\rho_j \\ \delta\dot{\rho}_j \end{bmatrix} = \begin{bmatrix} a_{x,j} & a_{y,j} & a_{z,j} & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & a_{x,j} & a_{y,j} & a_{z,j} & 0 & -1 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ \delta \dot{x} \\ \delta \dot{y} \\ \delta \dot{z} \\ c\delta t \\ c\delta \dot{t} \end{bmatrix} \quad (2)$$

In (2), the symbol δ denotes an error in a state. The receiver's Cartesian coordinates are represented by x, y, and z (dots above a value represent its time derivative). The receiver's clock error is denoted as t and the letter c represents the speed of light. The terms $a_{x,j}$, $a_{y,j}$, $a_{z,j}$ are the elements of a unit vector pointing from the receiver's estimated position to the *j-th* satellite.

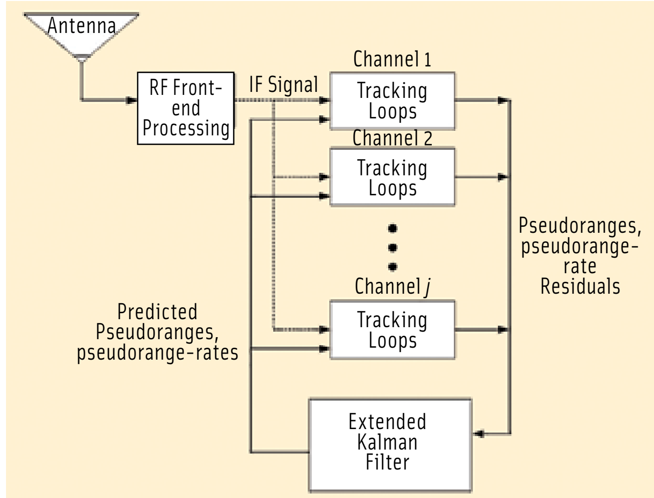

FIGURE 2 Vector Tracking Receiver Architecture

Equation (2) is very important in that it shows how the channels of the receiver are coupled. The pseudoranges are tied together through the three position states and one clock bias state. Similarly, the pseudorange-rates are coupled through three velocity states and one clock drift state. The position and velocity states are related to the residuals by the line-of-sight vectors.

We should note that the phase of the received carrier signals can also be tracked using the vector tracking approach. This is referred to as a vector phase lock loop (VPLL). The VPLL requires an alternate formulation of the central EKF due to the fact that the carrier phases of the received signals cannot be predicted unambiguously from the filter states shown in (1).

The VPLL is not as common as the VDLL and VFLL because the carrier frequencies and code phases can be tracked at lower C/N_0 ratios than the carrier phases. In general, vector tracking is used specifically for situations where low C/N_0 ratios are encountered.

The advantage of vector tracking over scalar tracking loops stems from the number of unknowns that the two algorithms are attempting to estimate, and how the unknowns are related to the available measurements. A traditional receiver uses N scalar DLLs to estimate N pseudoranges. In contrast, a VDLL uses N pseudorange residuals to estimate four states (three position and one clock bias). Similar numbers apply to the VFLL case as well and are therefore not provided here.

To illustrate this point, consider the situation where N pseudorange residual measurements are available, as shown in (3).

$$\begin{bmatrix}\delta\tilde{\rho}_1\\ \vdots\\ \delta\tilde{\rho}_N\end{bmatrix} = \begin{bmatrix}1&0&0&\cdots&0\\ 0&1&0&\cdots&0\\ \vdots&&\ddots&&\vdots\\ 0&&&&1\end{bmatrix}\cdot\begin{bmatrix}\delta\rho_1\\ \vdots\\ \delta\rho_N\end{bmatrix}+\begin{bmatrix}v_1\\ \vdots\\ v_N\end{bmatrix} \qquad (3)$$

$$\Delta\tilde{\rho} = I_{N\times N}\cdot\Delta\rho + V$$
$$E\{VV^T\} = \sigma_v^2\cdot I_{N\times N} = R_v$$

In this equation, the pseudorange residuals (denoted with a tilde) are assumed to consist of the true residuals plus white noise. In a manner analogous to using scalar DLLs, the pseudoranges are estimated using the equations in (3) with weighted least squares. The weighted least squares estimate of the pseudoranges ($\Delta\hat{\rho}$) and associated covariance are shown in (4).

$$\begin{aligned}\Delta\hat{\rho} &= (I_{N\times N}^T\cdot R_v^{-1}\cdot I_{N\times N})^{-1}\cdot I_{N\times N}^T\cdot R_v^{-1}\cdot\Delta\tilde{\rho}\\ &= \Delta\tilde{\rho}\end{aligned} \qquad (4)$$
$$\begin{aligned}E\{\Delta\hat{\rho}\,\Delta\hat{\rho}^T\} &= (I_{N\times N}^T\cdot R_v^{-1}\cdot I_{N\times N})^{-1}\\ &= \sigma_v^2\cdot I_{N\times N}\end{aligned}$$

Examining equation (4) reveals an important drawback of scalar tracking loops. As the number of available pseudoranges increases, the variance of the estimated pseudoranges remains constant. This is a direct result of the pseudoranges in (3) being modeled as completely uncoupled.

Now, consider using the N pseudorange residuals to first estimate three position errors and one clock bias error. This is analogous to the VDLL approach. Equation (5) relates the position and clock errors to the residuals.

$$\begin{bmatrix}\delta\tilde{\rho}_1\\ \vdots\\ \delta\tilde{\rho}_N\end{bmatrix} = \begin{bmatrix}a_{x,1}&a_{y,1}&a_{z,1}&1\\ \vdots&\vdots&\vdots&\vdots\\ a_{x,N}&a_{y,N}&a_{z,N}&1\end{bmatrix}\cdot\begin{bmatrix}\delta x\\ \delta y\\ \delta z\\ c\delta t\end{bmatrix}+\begin{bmatrix}v_1\\ \vdots\\ v_N\end{bmatrix} \qquad (5)$$

$$\Delta\tilde{\rho} = H_{N\times 4}\cdot\Delta X + V$$

The weighted least squares estimate of the vector ΔX and its associated covariance are shown in (6).

$$\Delta\hat{X} = (H_{N\times 4}^T\cdot R_v^{-1}\cdot H_{N\times 4})^{-1}\cdot H_{N\times 4}^T\cdot R_v^{-1}\cdot\Delta\tilde{\rho}$$
$$E\{\Delta\hat{X}\Delta\hat{X}^T\} = (H_{N\times 4}^T\cdot R_v^{-1}\cdot H_{N\times 4})^{-1} = \sigma_v^2\cdot(H_{N\times 4}^T\cdot H_{N\times 4})^{-1} \qquad (6)$$

The vector ΔX is related back to the estimated pseudoranges by Equation (7).

$$\Delta\hat{\rho} = H_{N\times 4}\cdot\Delta\hat{X} \qquad (7)$$

Therefore, the covariance of the estimated pseudoranges from the vector tracking approach are:

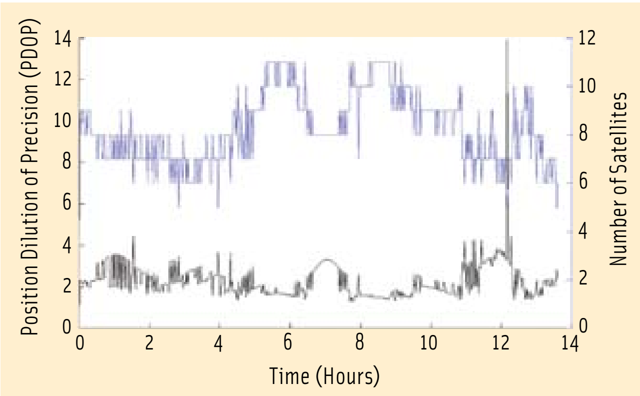

FIGURE 3 PDOP and Number of Satellites

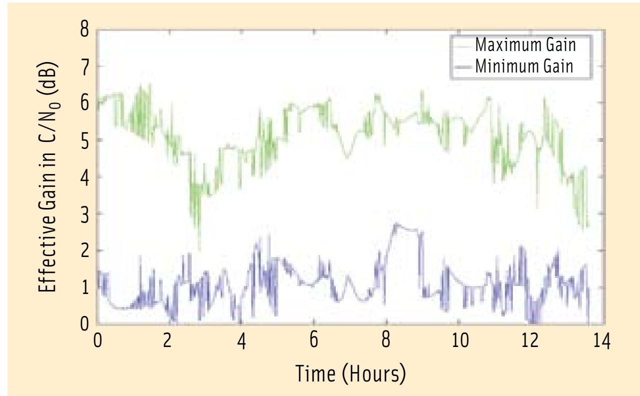

FIGURE 4 Maximum and Minimum Effective Gain from Vector Tracking

$$E\{\Delta\hat{\rho}\Delta\hat{\rho}^T\} = E\{H_{N\times4}\cdot\Delta\hat{X}\cdot\Delta\hat{X}^T\cdot H^T_{N\times4}\cdot\}$$

$$E\{\Delta\hat{\rho}\Delta\hat{\rho}^T\} = \sigma_v^2\cdot H_{N\times4}\cdot(H^T_{N\times4}\cdot H_{N\times4})^{-1}\cdot H^T_{N\times4} \qquad (8)$$

$$E\{\Delta\hat{\rho}\Delta\hat{\rho}^T\} = \sigma_v^2\cdot W_{NxN}$$

In other words, the variance of individual pseudoranges is determined by multiplying the appropriate diagonal element of the matrix W by the noise variance σ_v^2.

Comparing the pseudorange covariances in (4) and (8), the vector tracking approach will yield smaller pseudorange variances when the diagonal elements of W are less than one. In the case of four satellites, the pseudorange covariances in (4) and (8) are equal (assuming H has full rank).

In a case where N exceeds four, the pseudorange variances from the vector tracking method in (8) will generally be less than those in (4). This is the main benefit of vector-based tracking.

Equation (8) also shows that the performance of vector tracking is a function of how many satellites are available and their geometry. To determine the relative performance advantage of the vector-tracking algorithm for a typical GPS receiver, the visible satellite constellation was recorded every minute for about 14 hours at Auburn University.

For each satellite geometry, the effective gain in C/N_0 ratio was determined by examining the maximum and minimum diagonal elements of the matrix W in (8). A nominal C/N_0 ratio of 45 dB-Hz was assumed for all of the available satellites. At 45 dB-Hz, the noise variance σ_v^2 is 34.1 m^2.

The reduction in C/N_0 ratio needed to make the largest pseudorange variance equal to 34.1 m^2 is defined as the minimum gain in effective C/N_0 ratio. Conversely, the reduction in C/N_0 ratio needed to make the smallest pseudorange variance equal to 34.1 m^2 is defined as the maximum gain in effective C/N_0 ratio.

Figure 3 shows the position dilution of precision (PDOP) and number of visible satellites over the 14-hour period.

The maximum and minimum gain in effective C/N_0 ratio over the 14-hour period are shown in **Figure 4**.

The maximum gain in C/N_0 ratio varies from 2 to 6.5 decibels and has a mean of 5.1 decibels. The minimum gain in C/N_0 ratio varies from nearly 0 to 2.8 decibels and has a mean of 1.1 decibels. Figure 4 demonstrates that the vector approach can significantly improve a receiver's ability to track the received signals.

In conclusion, vector tracking algorithms combine the operations of signal tracking and navigation state estimation. The performance improvement brought about by vector tracking is contingent on the number of available satellites and their geometry. The only major drawbacks of vector tracking are their complexity and computational loads.

Mark Petovello is an Assistant Professor in the Department of Geomatics Engineering at the University of Calgary. He has been actively involved in many aspects of positioning and navigation since 1997 including GNSS algorithm development, inertial navigation, sensor integration, and software development.

Email: mark.petovello@ucalgary.ca

MATTHEW LASHLEY AND DAVID M. BEVLY

Matthew Lashley is a Ph.D. candidate at Auburn University. He is an employee of Navigation Technology Associates, Inc. and a member of the GPS and Vehicle Dynamics Lab (GAVLAB). His research interests are currently vector tracking and deep integration for GPS receivers.

David M. Bevly received his B.S. from Texas A&M University, an M.S. from Massachusetts Institute of Technology, and a Ph.D. from Stanford University in mechanical engineering. Bevly directs Auburn University's GPS and Vehicle Dynamics Laboratory (GAVLAB), which focuses on modeling, navigation, and control of vehicles.

SUGGESTED ARTICLES:

[1] Benson, D., "Interference Benefits of a Vector Delay Lock Loop (VDLL) GPS Receiver," in Proceedings of the 63rd Annual Meeting of the Institute of Navigation. Cambridge, Massachusetts, Institute of Navigation, April 2007

[2] Petovello, M., and G. Lachapelle, "Comparison of vector-based software receiver implementations with application to ultra-tight GPS/INS integration," in Proceedings of ION GNSS 2006. Fort Worth, Texas, Institute of Navigation, September 2006.

[3] Spilker, J. J., "Fundamentals of Signal Tracking Theory," in Global Positioning System: Theory and Applications, Vol. I. Progress in Astronautics and Aeronautics, Volume 163, AIAA, Washington, D.C., 1996 IG