

# GNSS Solutions:

## Single-versus Dual-Frequency Precise Point Positioning

“GNSS Solutions” is a regular column featuring questions and answers about technical aspects of GNSS. Readers are invited to send their questions to the columnist, Dr. Mark Petovello, Department of Geomatics Engineering, University of Calgary, who will find experts to answer them. His e-mail address can be found with his biography at the conclusion of the column.

### What are the tradeoffs between using L1-only and L1+L2 for PPP?

**P**recise point positioning (PPP) is a technique that can compute positions with a high accuracy anywhere on the globe using a single GNSS receiver. It relies on highly accurate satellite position and clock data that can be downloaded from the International GNSS Service (IGS) or obtained in real-time from a number of service providers, using either the Internet or satellite links.

As PPP does not rely on combining observations with simultaneous measurements from reference stations, it offers great operational flexibility and is extremely well suited for remote areas that lack the dense GNSS network infrastructure which we only find in cities and populated areas.

The best possible GNSS accuracy, a few centimeters or better, is obtained by using carrier phase measurements from dual-frequency receivers. However, single-frequency receivers can provide decimeter accuracy at a reduced cost for the receiver and generally reach this level of accuracy much faster than a dual-frequency receiver does.

A dual-frequency receiver may take as long as 20–40 minutes to reach centimeter accuracy. But a single-frequency receiver may give you decimeter accuracy within minutes. Indeed, single-frequency PPP may actually outperform the more advanced dual-frequency in terms of accuracy during the first 10 minutes or so . . . and yes, you read this correctly: single frequency is sometimes better than dual frequency.

Later in this article, we will explain the reasons for this surprising observation.

So then, what are the tradeoffs between single- and dual-frequency PPP?

A wide range of possible PPP algorithms exist, both for postprocessing applications as well as in real-time and with many variations to the correction models and processing strategies being used. Some of them are available commercially from different providers around the world; others are available free of charge.

Even more advanced approaches, including integer ambiguity resolution, are being investigated by research institutes and commercial organizations around the world and may come to the marketplace within a few years. It is impossible to describe all of these solutions here in any detail, only to give the basic operating principles.

The common denominator for all PPP solutions is that they rely on highly accurate satellite position and clock data, usually from an external service provider. The ultimate source for satellite position and clock data is the IGS, which can be downloaded from the Internet for free.

**Table 1** provides an overview of the satellite orbit and clock products available from IGS, together with the accuracy of the products. The IGS products with the highest accuracy are the IGS rapid and final products. However, these are not available in real-time and can only be used for postprocessing.

IGS also has predicted products, which are refreshed four times daily. Although these will not give you the same level of quality as with the rapid and final products, the solutions computed with the predicted part of the ultra-rapid orbits and clocks are still better than the stand-alone GPS solution. And, if you can wait for a few hours, you may also use the observed part of the ultra-rapid product, which has a much higher accuracy than the predicted part (See Table 1).

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For those who need the highest accuracy but cannot wait a few hours, or just for ease of use, observed orbit and clock data can also be made available in real-time. This is mainly the domain of a number of commercial providers and government agencies, such as the NASA/CalTech Jet Propulsion Lab, Veripos, Fugro, and so forth under various brand names.

Nowadays, a number of free, but experimental, real-time orbit and clock data streams are also available over the Internet. The main advantage of the commercial providers is that they provide users with the satellite position and orbit data over a satellite L-band link that is integrated with the receiver, so that you don't have to worry about obtaining the orbit and clock data yourself.

An important consideration with PPP is the necessity to correct for various effects that would otherwise cancel out in relative positioning techniques using one or more receivers. PPP needs to deal with many such effects, including site-displacements due to earth tides and ocean loading, phase wind-up, antenna phase center variations, and so forth. The higher the required accuracy, the more complicated the models need to be.

As we will discuss in more detail later, the main difference between L1 and dual-frequency implementations of PPP relate to how the atmospheric errors are handled. We begin by looking at the traditional dual-frequency processing approach before moving to the single-frequency case.

## Dual-Frequency PPP

Unlike with differential processing, in PPP the full effect of tropospheric delays must be estimated explicitly, resulting in an additional unknown *zenith tropospheric delay* (ZTD) parameter that must be estimated along with the position. Further, the standard approach in PPP for handling ionospheric delays is to use a dual-frequency receiver and form the “ion-

osphere-free” linear combination of L1 and L2 carrier phase and pseudorange observations:

$$\phi_{IF} = \alpha\phi_1 - (\alpha - 1)\phi_2 = \rho - c\delta t_r + d_{trop} + \lambda_{IF}[N_1 - \frac{1}{2}(\lambda_{WL}/\lambda_{IF} - 1)(N_2 - N_1)] + \varepsilon_{\phi_{IF}}$$

$$P_{IF} = \alpha P_1 - (\alpha - 1)P_2 = \rho - c\delta t_r + d_{trop} + \varepsilon_{P_{IF}}$$

with  $\alpha = f_1^2 / (f_1^2 - f_2^2) \approx 2.546$ ,  
 $\lambda_{IF} = c(f_1 - f_2) / (f_1^2 - f_2^2) \approx 10.6\text{cm}$   
 the ionosphere-free wavelength and  
 $\lambda_{WL} = c(f_1 + f_2) / (f_1^2 - f_2^2) \approx 86\text{cm}$  the wide-lane wavelength.

The given values are for the GPS L1 and L2 frequencies. These linear combinations fully eliminate the first-order ionosphere delay. Therefore, the parameters that the PPP algorithm has to solve for are: the position of the receiver, receiver clock error, ZTD, and the carrier phase ambiguities (or more precisely the ionosphere-free linear combination of carrier phase ambiguities corresponding to the part between rectangular brackets).

The ambiguity parameters remain constant over time — at least until the receiver loses the signal or a cycle slip occurs. The other parameters generally change over time. The receiver could be stationary, but more generally we assume that the receiver is in motion; so, the position will have to be estimated in kinematic mode. For the ZTD parameter a dynamic model is used to

link the estimates between epochs. The receiver clock error is estimated every epoch.

The PPP algorithm may use both the ionosphere-free linear combinations of carrier phase and pseudorange observations, or only the ionosphere-free linear combination of carrier phase observations. Using the pseudorange observations makes a difference for the initial convergence, but the final solution is almost fully dependent on the carrier phase data.

As mentioned earlier, dual-frequency position estimates take a long time to converge to centimeter- or decimeter-level accuracy, as illustrated by the example in **Figure 1**. The main reason for this slow convergence is that in a real-time operation the ambiguity parameters have to be estimated recursively. The ambiguities do not change over time, but at every epoch new and improved estimates of the ambiguities become available.

The accuracy of the ambiguity parameters is mainly influenced by the quality of the ionosphere-free linear combination of code data during the initial stages of convergence, but

	Latency	Updates	Sample interval	Orbit RMS	Clock RMS	St.Dev.
Broadcast	Real-time	--	daily	~100 cm	~5 ns	~75 cm
Ultra-Rapid (predicted half)	Real-time	03, 09, 15, 21 UTC	15 min	~5 cm	~3 ns	~45 cm
Ultra-Rapid (observed half)	3 - 9 hours	03, 09, 15, 21 UTC	15 min	~3 cm	~150 ps	~1.5 cm
Rapid	17 - 41 hours	17 UTC daily	15 min / 5 min	~2.5 cm	~75 ps	~7.5 mm
Final	12 - 18 days	every Thursday	15 min / 30s	~2.5 cm	~75 ps	~6 mm

TABLE 1. Accuracy and latency of IGS products <<http://igscb.jpl.nasa.gov/components/prods.html>>, retrieved June 26, 2012. Orbit accuracies are 1D mean RMS values over the three XYZ geocentric components; clock accuracies are expressed relative to the IGS timescale. The standard deviation (St.Dev.) values in the last column are clock standard deviation values converted into range units. These are computed by removing a separate bias for each satellite and station clock, whereas this is not done for the clock RMS values. The St.Dev. values are a more realistic representation of the ephemerides errors in PPP (which solve for the phase ambiguities) than the individual orbit and clock RMS. The first row, with the broadcast ephemerides, is not an IGS product and is only included for comparison.

after some time the changing satellite constellation and quality of the carrier phase data become the most important factors in achieving position accuracy.

For the example in Figure 1, only forward processing is used, i.e., for each epoch only the current and past measurements are used in order to show the convergence issue with real-time processing. Postprocessing offers another interesting option for kinematic processing: after the last epoch has been processed, the resulting ambiguities can be back-substituted (backward-processing) giving a more or less uniform precision over the whole data interval. In this case, no convergence issue appear, but the final precision depends on the length of the interval with uninterrupted data.

The ionosphere-free linear combination is very noisy as it amplifies multipath and receiver measurement errors. The ionosphere-free linear combination is also very costly in terms of redundancy: from four dual-frequency code and carrier phase observations, only two ionosphere-free observations remain.

In fact, if both the ionosphere-free linear combination of code and carrier phase data are formed, the ionosphere delay is eliminated twice (in other words, we estimate the ionosphere delay once from the code measurements and once from the carrier phase measurements). Typically, the noise in the ionosphere-free linear combinations would be a factor-three higher compared to the original observations. (See Mark Petovello's GNSS Solutions article in the January/February 2009 issue of *Inside GNSS* for details.)

With this in mind, convergence would be much faster if we did not have to use the ionosphere-free linear combination of code data. This becomes apparent with single-frequency PPP.

## Single-Frequency PPP

With single-frequency PPP we cannot form the ionosphere-free linear combinations introduced above. Broadly

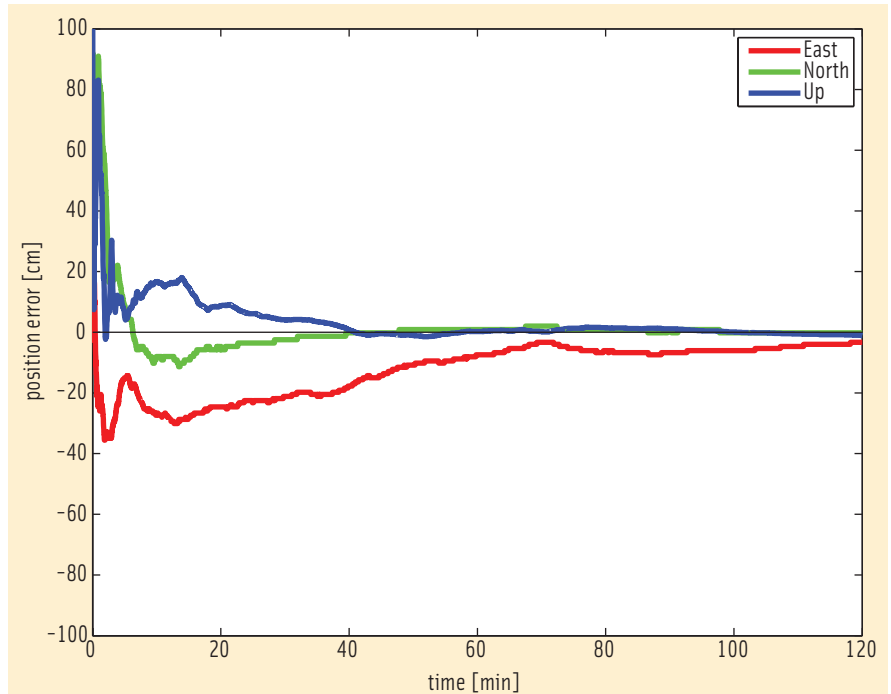


FIGURE 1 Dual-frequency PPP solution for a receiver in Delft, May 20, 2012, 0:00-2:00 (Final IGS orbits and clocks; static solution; forward processing only, no smoothing).

speaking, for single-frequency PPP there are two options

1. Use a linear combination of L1 code and carrier phase data, i.e.

$$\begin{aligned}\phi'_{IF} &= 0.5 * P_1 + 0.5 * \phi_1 = \\ &= \rho - c\delta t_r + d_{trop} + \frac{1}{2} \lambda_1 N_1 + \varepsilon_{\phi'_{IF}}\end{aligned}$$

which eliminates the ionosphere delay and halves the code noise.

2. Use ionosphere delays ( $I_{ext}$ ) from an external source, e.g. the gridded Global Ionosphere Maps (GIM) from IGS, to correct for ionosphere delay. This can be combined with carrier phase smoothing to obtain the following phase smoothed pseudorange

$$\begin{aligned}\bar{P}_{IF}[i] &= \frac{1}{i} (P_1[i] - I_{ext}[i]) + \frac{i-1}{i} (\bar{P}_{IF}[i-1] + \phi_1[i] - \phi_1[i-1] + I_{ext}[i] - I_{ext}[i-1]) \\ &= \rho - c\delta t_r + d_{trop} + \varepsilon_{\bar{P}_{IF}}\end{aligned}$$

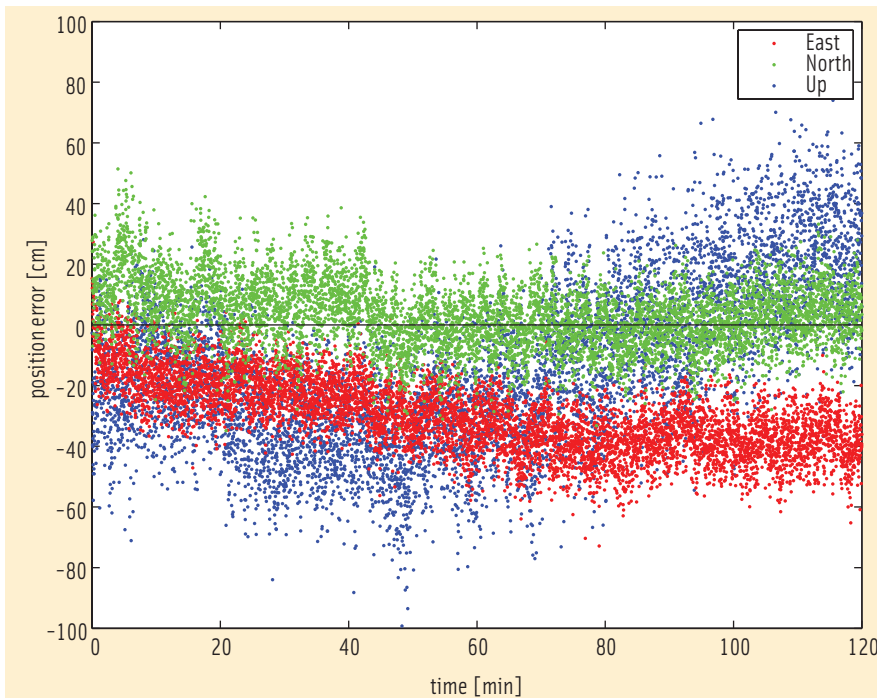
for epoch  $i = 1 \dots N$ . This expression is very similar to the expression for code-phase smoothing, whereby the external ionosphere model is introduced to overcome the problem of ionosphere divergence between the code and phase data. With the first option the vector

of unknown parameters consists of at least the receiver position, receiver clock, and carrier phase ambiguity. The main disadvantage of this option is that it requires us to estimate the carrier phase ambiguities. Therefore, the initial position estimates will not be very good, and convergence to the final accuracy will be slow.

The second option does *not* require us to solve for ambiguity parameters; the vector of unknown parameters consists only of receiver position and receiver clock. The ambiguity parameters are removed in the code-smoothing process. The convergence is much faster with this option.

For both options, we can estimate ZTDs as well, but this is often unnecessary because a model will provide sufficient accuracy.

In the second option the external ionosphere data actually takes on the role of the second frequency. This is illustrated in **Table 2**, which compares



**FIGURE 2** Single-frequency PPP solution for a receiver in Delft, The Netherlands, May 20, 2012, 0:00–2:00 (L1 C/A code and carrier phase; IGS Global Ionosphere Map, Final IGS orbits and clocks; kinematic solution; forward processing only).

tipath on L1 is amplified by a factor 2.546. Secondly, because of the techniques used to defy anti-spoofing, the noise on L2 pseudorange can be very large for some receivers, and also it involves a factor 1.546.

Finally, the TEC from IGS GIMs is computed from hundreds of receivers. And, despite the coarse models that are used for modelling the TEC in the GIM, the sheer amount of data that goes into these models makes them precise and very robust. However, we should point out the very limited experience with using these techniques under disturbed ionosphere conditions.

Given these factors, the single-frequency algorithm converges much faster than the dual-frequency algorithm, as illustrated in **Figure 2**. In Figure 1 and 2 the same dataset was used, but the processing options differed; for Figure 1 a static, carrier-phase dominated, dual-frequency solution is generated, and for Figure 2 a kinematic, single-frequency, pseudorange dominated solution is generated.

For many situations and under most ionosphere conditions, the L1-corrected pseudorange used by single-frequency PPP outperforms the dual-frequency ionosphere-free linear combination of pseudoranges. Because the convergence of dual-frequency PPP initially depends on the ionosphere-free linear combination of the code data, this explains why initially single-frequency PPP may outperform dual-frequency PPP. Of course, after some time, the ionosphere-free linear combination of carrier phase data kicks in

the standard deviation of the ionosphere-free linear combination of dual-frequency code data ( $P_{IF} = 2.546 * P_1 - 1.546 * P_2$ ) and single-frequency code data corrected for ionosphere delays ( $P_{IF} = P_1 - I_{ext}$ ).

Table 2 clearly shows that the ionosphere-corrected single-frequency observation often delivers superior standard deviations, especially for receivers with higher standard deviations for the pseudorange, and depending on the standard deviation of the ionosphere data. Typical standard deviations of IGS total electron count

(TEC) maps is 1–2 TEC units (TECUs) for well observed (mid-latitude) regions, whereas in polar, equatorial, and other under-observed regions TEC standard deviations can be much larger. The IGS website specifies a standard deviation for the TEC maps between 2–8 TECUs.

You may have asked yourself, “Why is the ionosphere-corrected pseudorange data better than the ionosphere-free linear combination?” The answer is, first of all, the factor 2.546 in the ionosphere-free linear combination, which means that both noise and mul-

$\sigma_{IF}$ (cm)		$\sigma_{P2}$ (cm)				
		10	20	30	50	80
$\sigma_{P1}$ (cm)	10	30	40	53	81	126
	20	53	60	69	93	134
	30	78	82	89	108	145
	50	130	131	135	149	177
	80	204	206	209	218	238

$\sigma_{IF}$ (cm)		$\sigma_{TEC}$ (TECU)				
		1	2	3	5	8
$\sigma_{P1}$ (cm)	10	19	34	49	81	128
	20	26	38	52	82	130
	30	34	44	57	85	131
	50	52	59	69	94	137
	80	82	86	93	113	151

**TABLE 2.** Accuracy of the ionosphere-free observations for the dual-frequency case using the ionosphere-free linear combination of L1 and L2 (left), and single-frequency case using TEC maps to correct the L1 pseudorange (right).

for dual-frequency PPP and becomes the determining factor for positioning accuracy.


Note that the IGS TEC map used for Figure 2 is not available in real-time. Although predicted IGS TEC maps are already available from the University of Berne today, a real-time TEC service is really what we need. Fortunately, experimental real-time TEC services are being developed by a number of institutes.

## Summary

So, for the final trade-off. We have shown that a single-frequency receiver may give you decimeter accuracy within minutes and actually outperform the more advanced dual frequency in terms of accuracy during the first 10 minutes or so. However, after some time, dual-frequency PPP will always outperform single-frequency PPP.

So, is dual-frequency PPP the winner?

Not necessarily. If cost is an issue, single frequency may win because the single-frequency PPP algorithm can also work with lower-cost receivers. Moreover, in cities, on highways, or in areas with lots of overhead structures and associated frequent loss of lock on GNSS signals, single-frequency PPP may outperform dual-frequency PPP.



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## Additional Resources

Gao, Y., "What is precise point positioning (PPP), and what are its requirements, advantages, and challenges?" *Inside GNSS*, November/December 2006

Several on-line services allow you to upload your data and will do the PPP processing for you, free of charge. Two examples of such services are the Automatic Precise Positioning Service from JPL <<http://apps.gdgps.net/>> (retrieved June 26, 2012) and NRCAN's Online Global GPS Processing Service <[http://www.geod.nrcan.gc.ca/products-produits/ppp\\_e.php](http://www.geod.nrcan.gc.ca/products-produits/ppp_e.php)> (retrieved June 26, 2012).

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