

Turning Point

Trajectory Analysis for Skiers

ADRIAN WÄGLI AND JAN SKALOUD

As with a growing number of athletes, competitive skiers are looking to GNSS technology to help evaluate and improve their performance. Positions logged during practice runs or races can be transformed into 3-D models of the exact trajectories skied to measure velocity, accelerations, and other performance variables. However, obstructions along a ski run, such as terrain and trees, can block satellite signals – suggesting a need to add low-cost inertial sensors to support continuous, accurate, and *affordable* positioning.

PHOTOS BY DAN FERRER, TWIB



Co-author Jan Skaloud passes a gate in a time trial of a prototype GPS/INS system for analyzing performance of skiers.

In many sports, the margin between victory and defeat may be a matter of a few hundredths of a second. Certainly that is true of skiing competition where the demands on equipment and the performance pressure on athletes are tremendous — and not just on elite skiers, but increasingly on participants at every skill level. In preparing for competition, every detail is important. Equipment is thoroughly tested and race preparation focuses on local factors (weather conditions, slope, qual-

ity of snow on the course, and so forth).

Traditionally, development and testing of materials or equipment has been based on repeated measurements with resources including timing cells or wind tunnels. Similarly, the analysis of athletes' performance has relied on techniques such as measuring race segments (chronometry) and video recordings.

These methods, however, appear vulnerable to changing meteorological conditions and the difficulty of replicating the posture and movements of test

subjects from one trial to the next due to such factors as improved performance stemming from cumulative experience in the trials or decreased performance due to fatigue. Furthermore, chronometry has a discrete character while researchers, coaches, and athletes are interested in observing certain phenomena continuously. Therefore, new methods are being sought that offer precise measurements during trials and subsequent evaluation of positions, velocities, accelerations, and forces.

Satellite-based positioning has already proven its effectiveness in many sports, including car racing and rowing. In particular, GNSS technology could bring its benefits to all disciplines in which the analysis of trajectory is crucial.

The continuous observation of the trajectory certainly has many advantages: the comparisons can be made over smaller sections (for example, gate-to-gate analysis in skiing) and can include topological aspects such as finding an

ideal trajectory by comparing different tracks. Furthermore, other parameters related to a defined section of the track (heart rates, velocities, respiration, etc.) can be compared rigorously.

Athletes and coaches are not only interested in the trajectories, but also in the motion analysis of segments of the human or the orientation of his equipment (lift-over of a motor cycle, torsion of skis, etc.). Current methods based on videogrammetry and timing cells, for instance, don't offer a flexible choice of

We've got an eye on the future

www.spirent.com/positioning
globalsales@spirent.com

For USA/Canada:
www.spirentfederal.com
info@spirentfederal.com



Engineer Stéphane Ducret, of TracEdge, calibrates the GPS reference receiver and mobile unit.

the intermediate times and course segments or the analysis of accelerations and velocities along the whole track.

In open spaces, GNSS supports continuous position, velocity, and acceleration analysis of racers' trajectories. The economic and environmental realities, however, are often far from such an ideal case. Considering the high dynamics of a skier and the ergonomic require-

ments placed on the equipment, today's technological limits in low-cost GNSS positioning are quickly reached or even exceeded.

Research discussed in the paper by J. Skaloud and P. Limpach, cited in the Additional Resource section at the end of this article, demonstrated that the hardware differences under actual conditions of skiing are much more decisive for precise positioning than the existing nuances between the ambiguity resolution algorithms.

For financial considerations the tracking of a large number of athletes

requires the use of low-cost, single-frequency (L1) GPS receivers. (The current pricing of dual-frequency GNSS receivers means that their use will be restricted to a few athletes and applications with high position-accuracy requirements.) Thus, the research presented here integrates L1 carrier-phase differential GPS algorithms that take into consideration the high dynamics of the athletes and their particular environment.

A skier's environment quickly alternates between open spaces and areas that block or attenuate satellite signals (sudden satellite masking), which makes resolution of the phase ambiguities difficult or even impossible. To overcome the lack of continuity of the GPS signals and in order to observe accelerations (and hence forces) directly, low-cost micro-electro-mechanical system (MEMS) inertial navigation units (IMUs) are integrated with GPS.

Such sensor combinations are suitable for this application because of their small size and limited cost. Also, the GPS/MEMS-IMU integration enables accurate determination of the orientation of a course segment and answers some of the questions raised earlier.

In this article, we introduce a method by which to model and compare trajectories in three dimensions. This modelling can be supported by quality indicators that evaluate the comparison's statistical significance. Then we present the integration of GPS and MEMS-IMU using an extended Kalman filter and assess its performance in a low-cost system we designed for trajectory analysis. Finally, we illustrate a comparison between traditional and GPS-based chronometry.

Trajectory Analysis

Analyses of athletic performance along curved and slightly varying courses or trajectories are often based on data sets that were recorded at different times. This situation occurs, for example, in downhill skiing competitions or training in which skiers make sequential (not simultaneous) runs.

Evaluation of trajectories may include the comparison of timing splits, their shape, velocities, and accelerations. Rel-

ative timing splits — whether between real or virtual gates — can be studied once the trajectories are known.

Such trajectories cannot be compared by considering only the differences in coordinates or velocities recorded at the same instant (as could be done for real-time comparison between two competitors). The left panel of **Figure 1** shows the trajectory of two athletes sampled at the same time interval. Obviously, athlete A is faster than B (the sampling points of athlete B are closer to each other).

Their velocity profiles with respect to the time from start are given in **Figure 1** (right). Based on the simple time-or-distance comparison, however, an accurate explanation could not be given for the substandard performance of athlete B and the points at which he lost time. In order to avoid such biased comparisons, the tracks need to be compared spatially and preferably in increments smaller than the intervals between gates on the course.

To compare trajectories accurately and efficiently, we model GPS or GPS/INS data sets as continuous curves (e.g. cubic splines). Then, we select a reference trajectory (e.g., athlete A because he is the fastest or the mathematical model of the optimal course), and compare all other trajectories by intersecting them with planes that are perpendicular to the trajectory of reference.

Figure 2 (left) shows a simplified schematic of a reference trajectory and a single trajectory that will be compared to it. Based on the intersection time of both trajectories with the plane, the difference between the athletes are computed.

Of course, we are looking first at coordinate differences between the trajectories, but we can also compare any other attributes attached to the trajectories (elapsed time, velocities, accelerations, heart rates, and so forth) in a straightforward way. Additional splits and (virtual) gates can be easily computed and interposed between the planes of the timing cells or gates already intersecting the track. Thanks to this modeling, the performance can be evaluated at any interval.

The spatial comparison enables us to

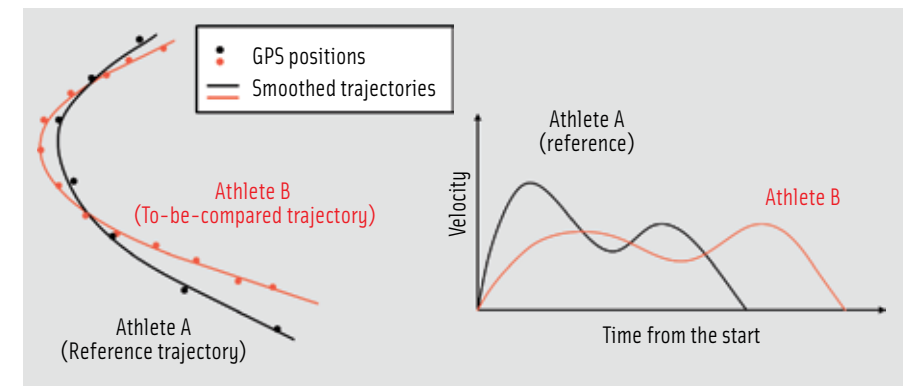


FIGURE 1 Trajectory and velocity evaluation based on distance or time comparison

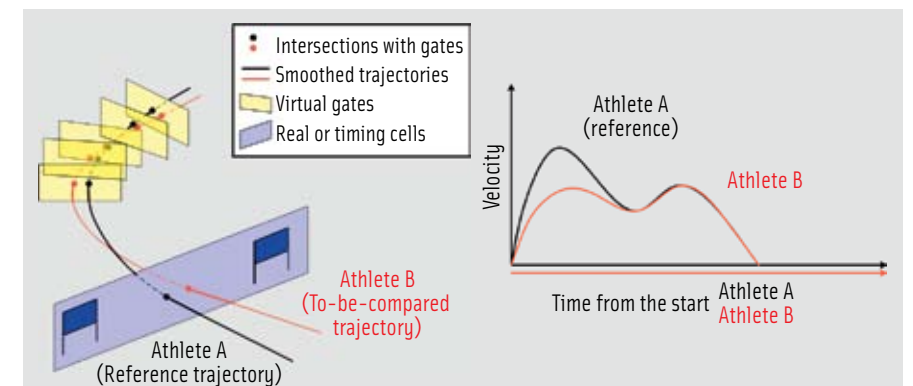


FIGURE 2 Principle of spatial trajectory comparison. The blue rectangle represents plane formed by a set of gates; yellow rectangles are virtual gates.

clearly conclude that the performance of athlete A in the first section of the track is largely superior to that of athlete B (**Figure 2**, right). In the second section, their performance is identical, as indicated by the closely overlapping, red and black trajectory lines. Alternatively, the abscissa could indicate the distance from the start and highlight different sections on the track (sectors, intermediates, gates, and so forth).

This methodology already has its commercial adaptation in a software package dedicated to the performance analysis in sport (**Figure 3**).

Quality Indicators

We can assess the varying navigation state accuracies by introducing a quality indicator (e.g., $\pm 3\sigma_d$), where σ_d is the

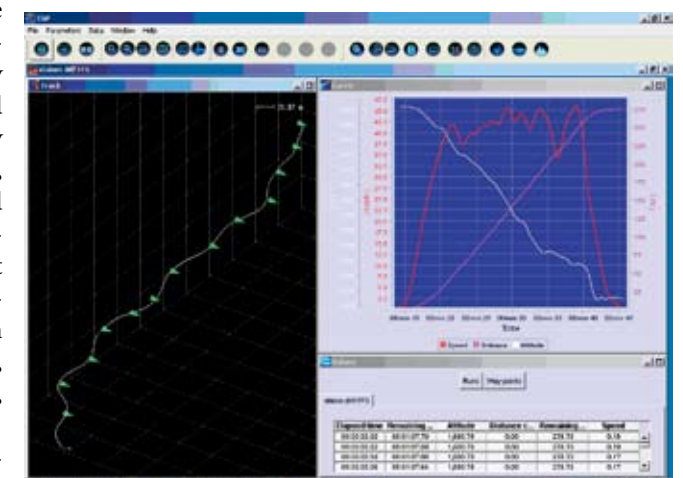


FIGURE 3 GPS chronometry displayed on the user interface of a commercial trajectory analysis system developed by TracEdge. The screen at left displays a trajectory modeled from positioning data logged in a run down a ski course. The screen on the right shows speed (red line), distance traveled (pink line), and altitude (white line) along various sections of the course. The table gives the numerical data and allows performing gate-to-gate comparisons.

standard deviation of the coordinate difference between two trajectories computed by error propagation based on the accuracies of both trajectories ($\sigma_{E,A}^2$, $\sigma_{E,B}^2$, $\sigma_{N,A}^2$, $\sigma_{N,B}^2$):

$$\sigma_d = \frac{1}{d} \sqrt{\Delta E^2 (\sigma_{E,A}^2 + \sigma_{E,B}^2) + \Delta N^2 (\sigma_{N,A}^2 + \sigma_{N,B}^2)}$$

Based on this indicator, rigorous conclusions about the significance of the performance differences can be made. This is illustrated for a trajectory comparison in **Figure 4** where the two trajectories can be considered as distinct only at sections with no overlap between the trajectory “snakes” formed by quality indicators.

GPS/MEMS-IMU Integration

The use of MEMS-IMU positioning to measure athletes’ performance is still in its early stages. The efforts described here began with research into the usual approach to GPS/INS integration in which inertial drifts and offsets are estimated by measurement of positions and velocities at predetermined reference points.

Given the context of high dynamics in sports and the quality of low-cost MEMS sensors, we wanted to verify whether the integration of inertial MEMS with GPS is feasible — especially considering the magnitude and change of their systematic errors and their sensitivity to temperature changes.

The paper by J. Skaloud and B. Merminod (see Additional Resources) suggested an approach based on a black-error model. A well-tuned model was identified as suitable for analyzing particular characteristics of the performance, but required a certain level of a priori knowledge of the underlying dynamics.

The previously mentioned research by J. Skaloud and P. Limpach tested the synergy of integrating a digital magnetic compass with GPS using a recursive QUEST algorithm. This approach required accelerations differentiated from GPS measurements as input for the attitude computation and thus meant that the derived inertial accelerations were not independent of GPS.

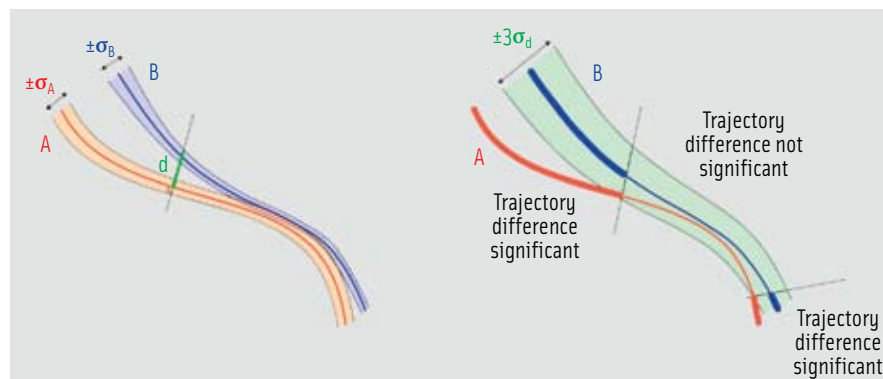


FIGURE 4 Quality indicator: Trajectories B is significantly different from trajectory A if it does not overlap the buffer of $\pm 3\sigma_d$ around trajectory B.

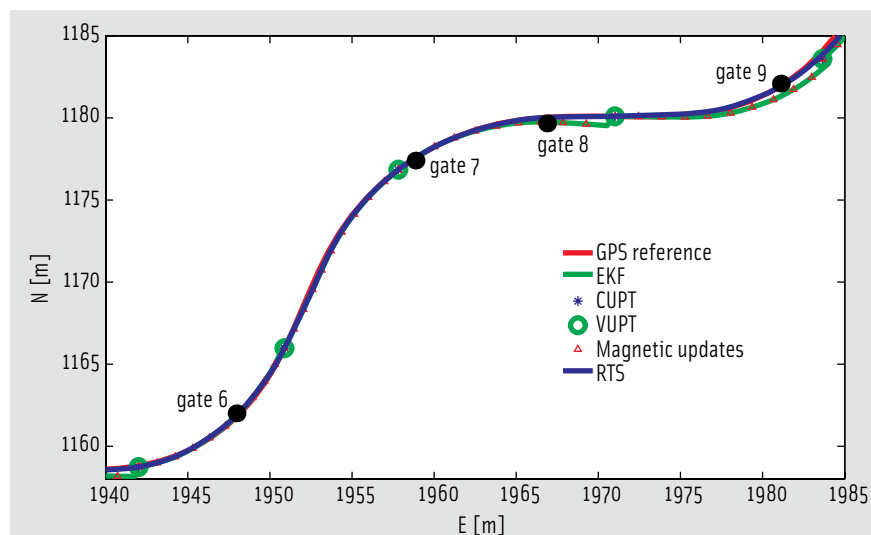


FIGURE 5 Loosely coupled GPS/MEMS-IMU integration with magnetic compensation. In red: The L1 GPS trajectory computed at 10Hz and smoothed with cubic splines. In green: The forward strap-down solution of the GPS/MEMS-IMU filter with position and velocity updates at 1Hz. In blue: The backward-smoothed RTS (Rauch-Tung-Striebel) solution. Coordinate updates (CUPT), velocity updates (VUPT) and magnetic updates are depicted with their respective symbols.

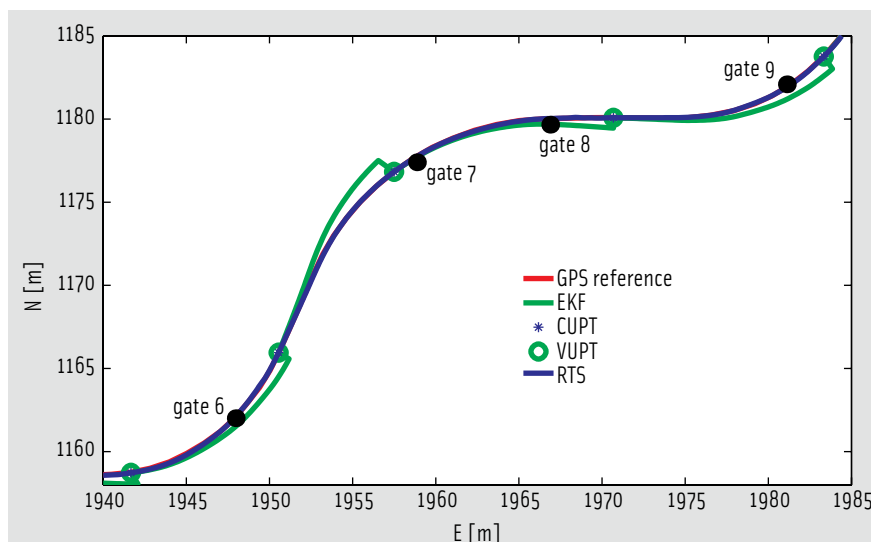


FIGURE 6 Loosely coupled GPS/MEMS-IMU integration without magnetic compensation.

Nonetheless, the results were sufficiently encouraging for further consideration of MEMS in sports.

Learning from this previous research, we decided to undertake a Kalman filter approach where the synergy of gyroscopes, magnetometers, and accelerometers would provide certain autonomy during the periods when GPS signals are obstructed.

In order to ascertain the best algorithm for this specific application, we implemented two GPS/MEMS-IMU integration strategies: a loosely coupled approach that integrated postprocessed GPS positions and velocities with the inertial measurements and, secondly, a closely coupled approach that input raw L1 GPS measurements directly into the extended Kalman filter (EKF).

The former design is more robust while the latter is more optimal and could prove advantageous in a skiing environment with frequent satellite blockages. (By optimal, we mean that statistically the Gaussian assumption seems more appropriate for pseudoranges and carrier phase measurements than for positions and velocities. Moreover, the second approach permits us to include GPS measurements even if a GPS position fix is not possible (e.g. $SV < 4$). The papers by K. P. Schwarz et al and B. Scherzinger, listed in Additional Resources, discuss the relative advantages of the two approaches in further detail.) The filters were implemented in the local-level frame to make the state interpretation straightforward.

For the inertial measurements, we considered a simplified model, judging that the misalignments, drifts, and constant offsets could not be decorrelated efficiently given the characteristics of the MEMS sensors. Hence, the inertial measurements are assumed to be affected only by a bias modeled as a first-order Gauss-Markov process:

$$\dot{\hat{l}}^b = l^b + \mathbf{b}_p + \mathbf{w}_p \text{ with}$$

$$\dot{\mathbf{b}}_p = -\beta \cdot \mathbf{b}_p + \sqrt{2\sigma_p \beta} \cdot \mathbf{w}_p$$

where \hat{l}^b is the estimated MEMS measurement (specific force, rotation rate, or magnetic field measurement), l^b the



Triple axis MEMS accelerometer, gyroscopes, and magnetometers placed on the skier's helmet

measured MEMS observation, \mathbf{b}_p the bias of the MEMS measurement, \mathbf{w}_p the measurement noise and β the inverse of the correlation time of the Gauss-Markov process.

A choice had also to be made with respect to the magnetometers. These sensors are useful for attitude estimation and thus indirectly help to bridge the gaps in GPS positioning. Unfortunately, they are prone to magnetic disturbances and their output is affected by high frequency accelerations.

Their measurements are introduced as external measurements using the following model:

$$h_m(\hat{\mathbf{x}}_k) = \mathbf{R}_n^b \mathbf{h}^n - \mathbf{b}_m + \mathbf{w}_m$$

where \mathbf{h}^n is the Earth's magnetic field vector for a specific location and time, \mathbf{b}_m the magnetic sensor bias expressed in the body frame, and \mathbf{R}_n^b the direction cosine matrix from the body frame to the navigation frame (local level frame).

We first validated the implemented algorithms with simulated MEMS-IMU measurements. For that, the MEMS error characteristics (noise, biases and drifts) were determined by static lab testing. Then, by grafting these errors into

the signals of a high quality IMU, we generated a “MEMS-IMU-like” data set that were then processed together with GPS data collected in the same test.

After successful validation, we tested the algorithms based on field experiments in — among other sports — Alpine skiing. The MEMS-IMU was placed on the skier's helmet underneath the GPS antenna, as seen in the accompanying photo. The GPS data was collected by a low-cost L1 receiver and an L1/L2 receiver for reference. The antennas for both receivers were mounted on the helmet.

Figure 5 and **Figure 6** show a section of two trajectories computed by loosely coupled GPS/MEMS-IMU integration with and without magnetic sensor, respectively. The gates of the giant slalom were determined with a static GPS survey and are plotted as external reference.

The measurement rate of the MEMS-IMU was 100Hz. GPS coordinate and velocity updates are input at a frequency of 1Hz, whereas magnetic updates are performed at 10Hz. It can be seen that the filtered IMU trajectory follows the skier's motion but starts diverging slightly after one second with a maximum error of half a meter.

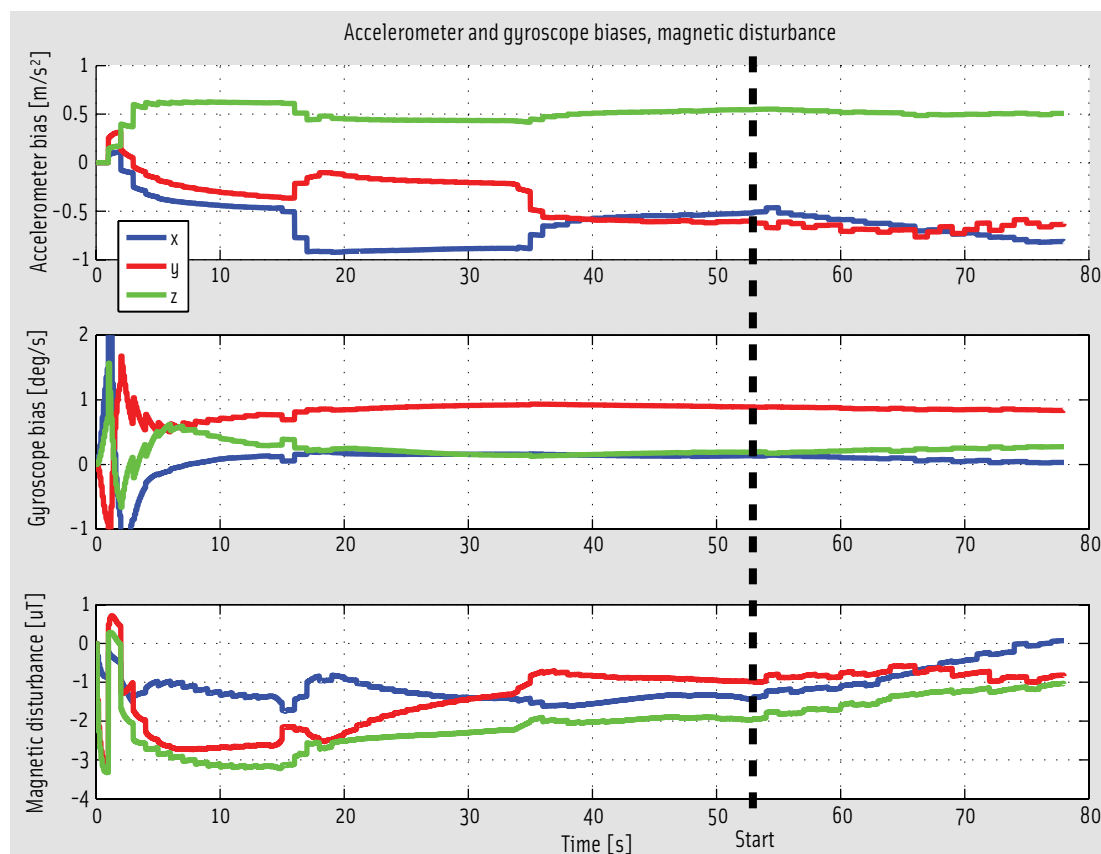


FIGURE 7 Estimation of the biases of the MEMS sensors. The biases converge rapidly after the start. The magnetic biases show a slight drift that might be real or caused by the high frequency accelerations.

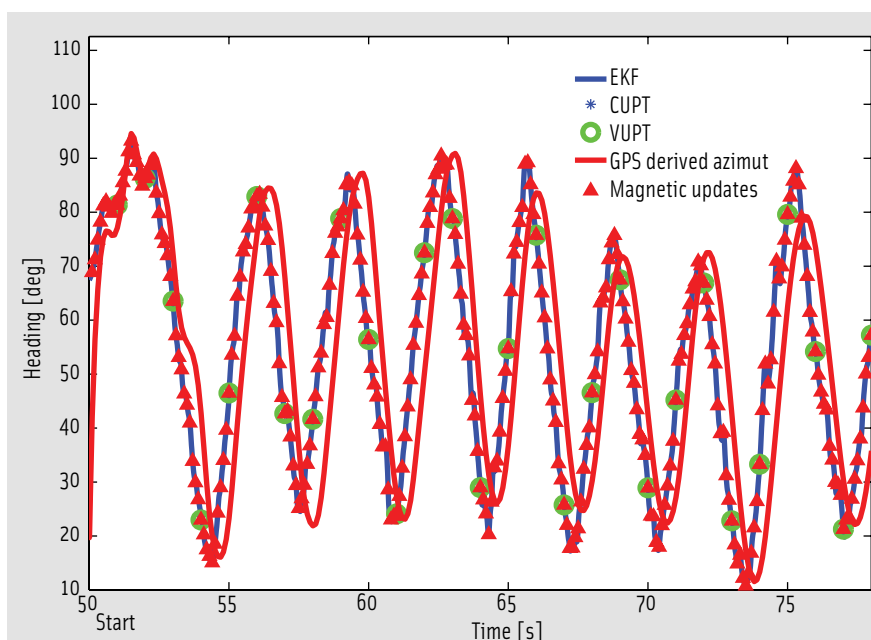


FIGURE 8 Estimation of the heading of the skiers' motion. The GPS derived heading describes the trajectories' orientation. The inertially determined heading represents the attitude of the MEMS sensor fixed on the skier's helmet. Hence, the two parameters can not be compared directly.

A backward-smoothing algorithm (Rauch-Tung-Striebel or RTS) compensates this divergence efficiently.

The resulting smoothed output nearly coincides with the reference trajectory computed with L1 GPS measurements at

10Hz and smoothed with cubic splines. A comparison between Figures 5 and 6 clearly shows that the magnetic sensors improve the attitude estimation considerably in this experiment.

On other sections of the course the performance enhancement is less obvious — probably because of the magnetic sensors' sensitivity to high frequency accelerations. Therefore, we plan to assess the contribution of the magnetometers in a new set of experiments where the reference signals are also present in the orientation domain.

Figure 7 shows an example of the convergence of the modeled biases in the MEMS data. The filter converges rapidly, which is a crucial factor in sports application where fast adaptation to the systematic errors is expected due to the changing dynamics (e.g., after the start of a race). While the gyroscope biases remain stable, the magnetic biases seem to be affected by the accelerations during the ski run. (Incidentally, this confirms the work of D. Törnqvist mentioned in Additional Resources, which noted the sensitivity of magnetic sensors to high frequency accelerations.)

Figure 8 illustrates the MEMS sensor attitude during the run, which reflects the movement and orientation of the helmet (and head) of the skier at discrete points over the course based on the GPS/MEMS integration. Again, the filter converges rapidly after the start of the run. The GPS-derived heading (azimuth of the tangent to the GPS trajectory) indicates the overall motion of the skier.

Because the GPS and MEMS data reflect the orientation of two distinct elements, they cannot be directly com-

pared. However, comparison of the MEMS data with high-accuracy inertial data will help to validate the computed orientation of the helmet by the MEMS sensors.

The same trajectory was also computed in closely coupled mode. We introduced raw L1 GPS measurements at a frequency of 1Hz. This calculation confirmed that the magnetic sensors improve the trajectory estimation. Compared to the loosely coupled approach, the RTS-smoothed trajectory has small differences to the reference GPS trajectory (maximum error of 30 centimeters).

These data, however, were collected under a favorable satellite constellation (number of visible satellites, low position dilution of precision or PDOP). Therefore, we need to pursue additional simulations of effects caused by successively excluded satellites as well as using data collected in adverse satellite-tracking conditions in order to reach a conclusion about the relative performance of the two integration strategies.

Trajectory Comparison

The following example illustrates the performance analysis based on the algorithms for trajectory comparison described earlier in this article. The data presented here were collected by a skier equipped with a low-cost L1 GPS receiver and a triple-axis MEMS-IMU.

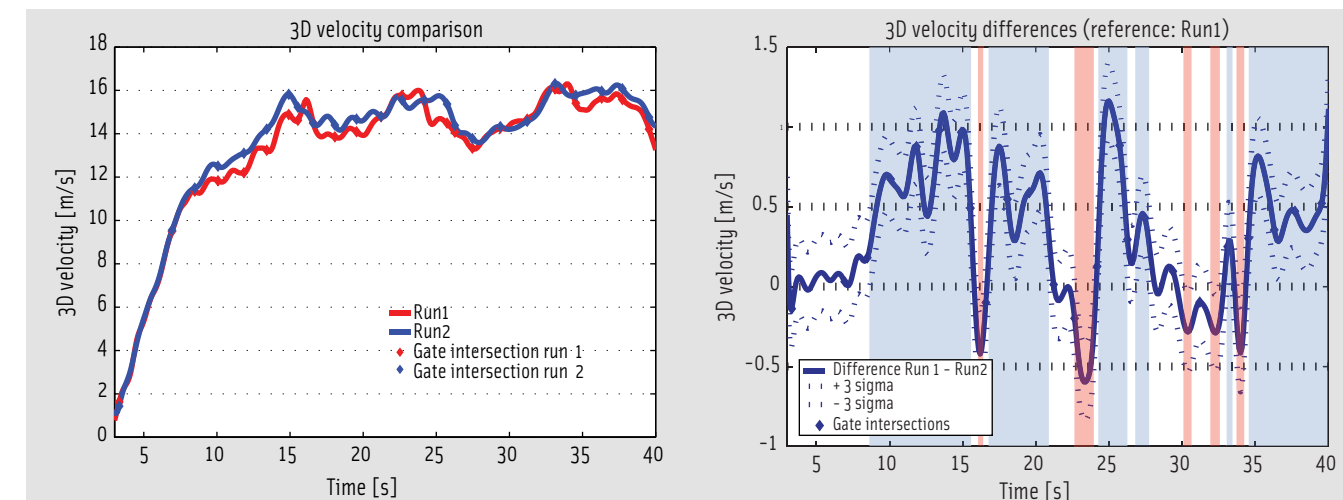


FIGURE 9 Velocity comparison based on the spatial comparison algorithms and illustration of the accuracy indicator. The shaded areas indicate where the skiers' velocity difference was significant.



Stephane Ducret (left) and EPFL graduate student Jean-Marie Bonnaz (right), adjust GPS/INS unit before ski run by Pierre Ribot, of TracEdge.

Figure 9 shows the velocity profiles of two runs in a giant slalom and their respective differences augmented by a quality indicator (± 3 sigma). Gate intersections are plotted to provide an external reference. Based on this figure, the performance of the skier during the two runs can be evaluated gate by gate, but only at times where such difference is marked as significant.

Of course, GPS-based trajectory computation can be applied to many other areas of performance analysis. The sidebar entitled "Drifting Tires" describes its potential use to measure the amount of side-slip experienced by a racing motorcycle.

Chronometry Past and Future

A frequently asked question in the sports domain is the accuracy of GPS chronometry compared to the traditional approach based on timing cells. The following investigation, undertaken in collaboration with TracEdge and the German ski federation (DSV), demonstrates the feasibility of GPS chronometry and illustrates its advantages over the classical approach.

A professional downhill skier was equipped with an L1/L2 GPS receiver. He performed 10 super-G-like runs alternating two pairs of skis (Number 29 and 101). The purpose of the test was

to determine the fastest ski. Three timing splits were measured (start, intermediate, and arrival) using a professional timing system.

The timing gate locations were determined in postprocessing based on L1/L2 measurements with sub-decimeter accuracy. Intersecting the GPS trajectories with the timing gates as depicted in Figure 2 allows us to determine GPS intermediates, which can then be compared to those of the timing cells.

Table 1 presents the GPS derived intermediates and splits derived from timing cells collected. Both methods identify ski number 101 to be the fastest.

The standard deviation of the differences between the two sets of skis is similar for both methods. Hence, both methods provide the same result with similar accuracies. The advantage of GPS chronometry over the classical approach is that virtual splits can be introduced which allow refining the evaluation of

the skis depending on the slope, wind or snow conditions.

The difference between the individual splits derived by GPS and splits derived from timing cells amounts to 3-7 hundreds of a second. How can this discrepancy be explained? A constant offset can be explained by the accuracy of the gate coordinates. However, varying differences are caused by numerous factors.

First, the carrier-phase ambiguities could only be fixed during the first five runs due to adverse satellite conditions with northern exposure and a slope bounded by woods, which explains the random differences during the last five runs. The varying accuracy of GPS with float ambiguities is a major error source: A positioning error of 50 centimeters at 60km/h results in a timing error of 5 hundreds of a second.

Second, the fact that the GPS receiver is placed on the helmet whereas the tim-

ing cells are intersected by the skier's feet or hands is a negligible error source: A longitudinal change in position of the skier's head (and GPS-equipped helmet) with respect to his feet of 20 centimeters will cause a timing error of 1.2 hundreds of a second. However, if he intersects the split with his hands, the difference could become significant and might explain certain outliers.

Unfortunately, the chronometry system based on the timing cells used in the test was not certified, and, therefore, no assertion about its timing accuracy can be made. An additional test using GPS-synchronized chronometers would need to be performed in order to investigate the observed differences.

Nevertheless, the GPS-based determination of the fastest ski provides the same result with a similar accuracy as the one derived from the timing cells. This demonstrates that GPS chronometry is an interesting alternative, even in a

| Ski | Start-Intermediate | | Difference | Start-Arrival | | Difference | Ambiguity Status |
|---|--------------------|---------|-------------------|---------------|---------|-------------------|------------------|
| | Cell [s] | GPS [s] | cell-GPS [1/100s] | Cell [s] | GPS [s] | cell-GPS [1/100s] | |
| 29 | 16.54 | 16.62 | -8 | 36.73 | 36.76 | -3 | F |
| 101 | 16.62 | 16.70 | -8 | 36.75 | 36.78 | -3 | F |
| 29 | 16.70 | 16.75 | -5 | 36.68 | 36.72 | -4 | F |
| 101 | 16.40 | 16.48 | -8 | 36.39 | 36.42 | -3 | F |
| 29 | 16.61 | 16.67 | -6 | 36.66 | 36.68 | -2 | F |
| 101 | 16.44 | 16.56 | -13 | 36.44 | 36.51 | -6 | L |
| 29 | 16.58 | 16.59 | -1 | 36.82 | 36.71 | 12 | L |
| 101 | 16.45 | 16.44 | 1 | 36.55 | 36.45 | 9 | L |
| 29 | 16.75 | 16.92 | -17 | 37.45 | 37.61 | -15 | L |
| 101 | 17.00 | 17.10 | -10 | 37.90 | 37.91 | 0 | L |
| Mean [s] | | | | | | | |
| 29 | 16.63 | 16.71 | 4.0 | 36.87 | 36.90 | 6.1 | |
| 101 | 16.58 | 16.66 | 7.2 | 36.81 | 36.81 | 6.2 | |
| Difference ski 29-101 [1/100s] | | | | | | | |
| | 5 | 5 | | 6 | 8 | | |
| Standard deviation [1/100s] | | | | | | | |
| 29 | 9 | 13 | | 33 | 40 | | |
| 101 | 25 | 27 | | 63 | 63 | | |
| Mean difference GPS-Cell [1/100s] | | | | | | | |
| | | | -7 | | | -2 | |
| Standard deviation difference GPS-Cell [1/100s] | | | | | | | |
| | | | 5 | | | 8 | |

TABLE 1. Comparison between classical chronometry and GPS chronometry



Jan Skaloud and GPS/INS-equipped helmet

difficult environment, that offers additional flexibility for evaluating performance. We also experienced that the splits derived from L1 GPS data are only negligibly noisier than those derived from L1/L2 data and lead to the same conclusion for the ski selection.

Perspectives

We described the requirements for performance analysis in sports by highlighting typical applications like trajectory comparison, chronometry

and drift computation based on GPS-augmented data.

The presented approach based on low-cost GPS/MEMS integration provides interesting results for the performance analysis of athletes with body-worn sensors as it fulfills the requirements for most sports applications for in terms of accuracy, ergonomy and cost. L1/L2 receivers are required for higher accuracy needs but will be reserved to few applications. Nevertheless, further tests are required to validate and improve the MEMS-IMU error model and to evaluate the need for closely coupled integration in conditions with difficult satellite reception. The convergence criterion is very important in sports, where the filter has to adapt rapidly to the changing dynamics. Thus, other integration strategies will be tested in order to better cope with this aspect.

The trajectory comparison approach has proven to be very efficient for performance evaluation in sports. This was

Precision OEM Modules

Hemisphere GPS pioneered the development of affordable precision GPS technology for a variety of commercial and industrial applications.

Our board-level products include:

- Crescent® GPS**
High-performance GPS and SBAS receiver module features optional e-Dif® and RTK differential solutions and high update rates.
- Crescent Vector®**
GPS compass with dual-antenna input supports accurate heading and positioning on a single board.
- SBX-4**
Dual channel radiobeacon receiver module. Compliant to latest IEC 61108-4.
- LX-1**
OmniSTAR receiver module directly compatible with Crescent GPS.

With a long history of successful OEM partnerships in demanding markets, we have the right solution for your next precision GPS design. Contact us today.

www.hemispheregps.com • precision@hemispheregps.com

Drifting Tires

An important aspect in the performance assessment of tires is their side slipping drift. This drift is defined as the angular difference between the direction of motion of the wheel and the direction of motion of the vehicle (See Figure 1). The following example describes how the drift of the back wheels of a sports car was determined using GPS measurements and subsequent analysis with quality indicators.

Two L1/L2 GPS receivers were mounted on a sports car, one at the top of the axle on the back wheels and the second in the front part. Modelling the two GPS-derived trajectories with cubic splines, the computation of the direction of motion of the two receivers becomes straightforward because the drift can be expressed as an angular difference of the two respective trajectory tangents.

Figure 1 illustrates the derivation of the drift for the test car. During sharp turns, the deduced drifts of the tires become significantly different from zero. However, at long continuous turns, the value of the drift is reduced and its accuracy does not allow quantifying it as significant.

Note, that the drifts computed based on L1 GPS data did not significantly differ from those computed with L1/L2 data as long as the ambiguities can be fixed. This restricts the use of the L1 approach to shorter baselines.

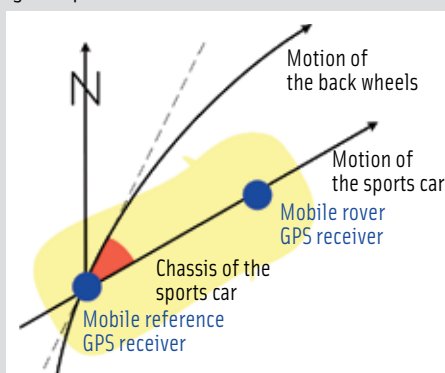


FIGURE 1 Drift of a car's tires: During sharp turns, the indicated confidence level does not include zero. Thus, the drifts of the back wheels become significant. During smoother turns or in straight-line movement, the drift is considerably smaller and cannot be characterized as statistically significant.

illustrated for trajectory comparison between athletes or for material selection. The material testing based on GPS chronometry presents an interesting alternative to the rather inflexible approach based on timing cells. The inclusion of accuracy indicators is indispensable to evaluate the significance of the to-be-compared parameters.

Acknowledgement

The authors greatly appreciate the support of Pierre Ribot from TracEdge, Charly Waibel from the German Ski Federation DSV, and Andreas Huber of the Olympiastützpunkt Bayern for providing us the timing splits of the present ski experiment. This work is financed by TracEdge based at Chambéry, France.

Manufacturers

TracEdge, based in Le Bourget du Lac, France, has developed a GPS receiver for athletes and coaches based on DG16 boards from **Thales Navigation** (now **Magellan GPS**, Santa Clara, California, USA). TracEdge's performance analysis software TSP implements the algorithms developed in the Trajectory Analysis section of this article. The high-accu-

racy GPS reference system employed the dual-frequency Legacy GPS/GLONASS receiver developed by **Javad Positioning Systems** (now **Javad Navigation Systems**, San Jose, California, USA). The low-cost GPS system is based on an TIM-LL receiver from **u-blox AG** (Thalwil, Switzerland) coupled with a MEMS inertial sensor (3DM-G) from **Microstrain** Williston, Vermont, USA. An ALGE timing system combined with data transmission through Motorola radios was used for the GPS chronometry investigation. The reference trajectory was processed with Waypoint postprocessing software GrafNav from **NovAtel**, Calgary, Alberta, Canada.

Additional Resources

- [1] Bar-Itzhack, I. Y., "REQUEST - A New Recursive Algorithm for Attitude Determination," *NASA Goddard Space Flight Center*.
- [2] How, J., and N. Pohlman, and C.-W. Park, "GPS Estimation Algorithms for Precise Velocity, Slip and Race-track Position Measurements," in *SAE Motorsports Engineering Conference & Exhibition*, 2002.
- [3] Nachbauer, W., and P. Kaps, B. M. Nigg, F. Brunner, A. Lutz, G. Obkircher, and M. Mössner, "A video technique for obtaining 3-D coordinates in

Alpine skiing," *Journal of Applied Biomechanics*, pp. 104-115, 1996.

[4] Scherzinger, B., "Precise Robust Positioning with Inertially Aided RTK," *Journal of The Institute of Navigation*, vol. 53, Summer 2006.

[5] Schwarz, K. P., and M. Wei, and M. Van Gelderen, "Aided Versus Embedded - A comparison of two approaches to GPS/INS integration."

[6] Skaloud, J., and P. Limpach, "Synergy of CP-DGPS, Accelerometry and Magnetic Sensors for Precise Trajectory in Ski Racing," in *ION GPS/GNSS 2003*, Portland, 2003.

[7] Skaloud, J., and B. Merminod, "DGPS-Calibrated Accelerometric System for Dynamic Sports Events," in *ION GPS*, Salt Lake City, 2000.

[8] Titterton, D. H., and J. L. Weston, *Strapdown Inertial Navigation Technology*, Peter Peregrinus Ltd., 1997.

[9] Törnqvist, D., "Statistical Fault Detection with Applications to IMU Disturbances," in *Department of Electrical Engineering*, Linköping, Sweden, 2006.

[10] Zhang, K., and R. Grenfell, R. Deakin, Y. Li, Jason Zhang, A. Hahn, C. Gore, and T. Rice, "Towards a Low-Cost, High Output Rate, Real-Time GPS Rowing Coaching and Training System," in *ION GNSS 2003:16th International Technical Meeting of the Satellite Division of The Institute of Navigation*, Portland, 2003.

Authors



Adrian Wägli obtained a M.Sc. in geomatics engineering from EPF Lausanne, Switzerland, for his work on the ionospheric corrections of EGNOS. After working as a surveyor for the Swiss Federal Office of Topography, he joined the Geodetic Engineering Laboratory of EPFL in 2004 as a Ph.D. student. His research focuses on the integration of satellite and inertial navigation systems for performance analysis in sports.



Jan Skaloud is a scientist and lecturer at the Institute of Geomatics at EPF Lausanne, Switzerland. He holds a Ph.D. and M.Sc. in geomatics engineering from the University of Calgary and Dipl. Ing. in surveying engineering from the Czech Institute of Technology, Prague. He has been involved with the GPS research and development since 1993 and has worked extensively on the integration of GPS and inertial navigation systems for precise airborne and terrestrial mapping. 